Problem 9

Use the preliminary test to decide whether the following series are divergent or require further testing. *Careful:* Do *not* say that a series is convergent; the preliminary test cannot decide this.

$$\sum_{n=1}^{\infty} \frac{3^n}{2^n + 3^n}$$

Solution

Take the limit of the summand as $n \to \infty$.

$$\lim_{n\to\infty} \frac{3^n}{2^n + 3^n} = \lim_{n\to\infty} \frac{1}{\frac{2^n}{3^n} + 1}$$

$$= \lim_{n\to\infty} \frac{1}{\left(\frac{2}{3}\right)^n + 1}$$

$$= \lim_{n\to\infty} \frac{1}{\exp\left[\ln\left(\frac{2}{3}\right)^n\right] + 1}$$

$$= \lim_{n\to\infty} \frac{1}{\exp\left[n\ln\left(\frac{2}{3}\right)\right] + 1}$$

$$= \lim_{n\to\infty} \frac{1}{\exp\left[-n\ln\left(\frac{3}{2}\right)\right] + 1}$$

$$= \frac{1}{0 + 1}$$

$$= 1$$

Since it's not zero, the series diverges by the preliminary test.